2026/2027 SOUTHERN CALIFORNIA REGIONAL INTERNATIONAL COLLEGIATE PROGRAMMING CONTEST

Problem 6
Probabilistic Descent

We are given an integer n ($1 < n \leq 18$) and a perfect binary tree of depth $n-1$. The binary tree has exactly $2^n - 1$ vertices (so each leaf has depth $n-1$), the root has number one and each vertex i has left child $2i$ and right child $2i+1$. In each vertex, there is a number x_i, $0 < x_i < 2^n$ (except $x_1 = 0$).

We are going to play a game on the given binary tree. The object of the game is to maximize the total score by descending the tree one or more times. The total score starts at zero. Points start at zero and are accumulated by making moves that descend the tree.

- In order to move, a random integer r is drawn between 0 and $2^n - 1$ inclusive. The player can move to any child where $x_i \wedge r = 0$. ($\wedge$ is the bit-wise AND operator.) Moving to child i adds x_i points to the descent.
- Before each move, the player can choose to end the game instead and add the points accumulated from the current descent to the total score.
- Should the player attempt to move, but the random number drawn is such that neither child can be reached, the game ends immediately. No points are received for the current descent and the total score is final.
- If one of the tree's leaf nodes is reached, all the accumulated points for the descent are added to the total score. The player can end the game or start another descent back at the root with points reset to zero.

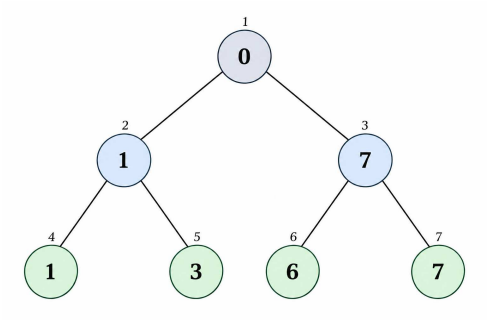


Figure 1. The binary tree from Sample Input 2.

Assuming optimal plays are made, what is the expected ending score?

The first line of input contains the integer n, $1 < n \leq 18$. The second line of input has $2^n - 1$ numbers, separated by spaces, where the ith integer is x_i.

Output the expected ending score if you play the game optimally. Your answer will be accepted if the relative or absolute difference from the judges' reference value does not exceed 10^{-5}.

Sample Input 1

```
2
0 1 2
```

Output for Sample Input 1

```
5
```

Problem 6
Probabilistic Descent (continued)

Sample Input 2

```
3
0 1 7 1 3 6 7
```

Output for Sample Input 2

```
1.7692307694
```

Sample Input 3

```
4
0 1 1 1 1 1 1 1 1 1 1 1 1 1 1
```

Output for Sample Input 3

```
0.5
```